Equilibrium real interest rates in the US are negative 2%
Estimates based on the modified LW model and its implications

< Summary >

◆ The secular stagnation theory set forth by former US Secretary of the Treasury Lawrence Summers has attracted attention to the idea of equilibrium real interest rates. This theory postulates that equilibrium real interest rates have declined to a substantially low level, and that conventional monetary easing measures are no longer effective as a result.

◆ According to a seminal paper and their updates by Laubach and Williams (2003, 2016), equilibrium real interest rates have been falling, but are currently hovering at nearly zero percent. Thus their model (LW model) does not strongly support the secular stagnation story.

◆ However, upon the incorporation of ex-post inflation expectation into the LW model, our new estimates show that equilibrium real interest rates have reached negative 2%. This result implies the validity of the secular stagnation hypothesis.
This report aims to critically review the author’s earlier June 2014 report, *Mizuho Economic Outlook and Analysis: “Secular stagnation theory” and the fall in natural interest rates – long-term revision of policy interest rates may run the risk of excessive monetary easing.*\(^1\) In the previous report, the author introduced Mr. Summers’ secular stagnation hypothesis, which stated that equilibrium real interest rates (his report uses the term “natural interest rates”) had fallen significantly, as well as a skeptical view of this theory by former Chairman of the Federal Reserve Benjamin Bernanke.

Our latest estimation results, however, suggest that a significant decline in equilibrium real interest rates may be possible, and it is highly likely that Mr. Summers’ secular stagnation theory will become a reality.

1. **Secular stagnation theory by Mr. Summers**

Mr. Summers’ secular stagnation theory assumes that equilibrium real interest rates are in significant negative territory, and therefore, conventional monetary policy accommodation, or controlling policy interest rates lower, is not sufficient to stimulate the economy.

According to Mr. Summers, the term “secular stagnation” refers to the situation where “the economy may not easily reequilibrate,”\(^2\) and it may take more time than anticipated to recover the state of equilibrium. He states that in the mid-2000s before the outbreak of the financial crisis, the US economy may have already faced a serious demand shortage. If we look back on the period when the US economy was beginning to experience a serious credit bubble under a loose monetary environment, no excess demand was generated and the inflation rate remained stable. In other words, “even a great bubble wasn’t enough to produce any excess in aggregate demand.”\(^3\)

Coexistence of the bubble economy and full employment in the late 2000s, as well as the lingering demand shortage, drove Mr. Summers to conclude that “equilibrium real interest rates in the US had fallen to the range of negative 2 or negative 3 percent in the mid-2000s,”\(^4\) as this would explain the whole picture.

Whether the monetary environment is loose or tight is determined by the difference between real interest rates and equilibrium real interest rates. Real interest rates can be calculated by “subtracting the expected inflation rate from the nominal interest rate.” On

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\(^1\) The report’s web address is as follows: http://www.mizuho-ri.co.jp/publication/research/pdf/insight/us140609.pdf


\(^4\) Remarks by Summers (2013), “Suppose the short-term real interest that was consistent with full employment had fallen to negative 2 or negative 3 percent sometime in the middle of the last decade.”
the other hand, according to Mr. Summers, equilibrium interest rates adjust to balance the supply of savings and demand for investment in the economy, just as the price of goods adjusts to balance the supply and demand for goods. So, the interest rates that balance savings and investment at full employment are the equilibrium real interest rates (or natural or neutral interest rates). Secular stagnation occurs when equilibrium real interest rates are sufficiently low that a loose monetary environment cannot be achieved through conventional central bank policies. Under such circumstances, the desired level of savings exceeds the desired level of investment, leading to a shortfall in demand and stunted growth.

2. Current status of equilibrium real interest rates

As Mr. Summers points out, for secular stagnation to be a valid hypothesis, one needs to show that equilibrium real interest rates have declined to an abnormally low level. But according to the most famous estimation results, equilibrium real interest rates have not fallen sufficiently to serve as solid evidence to support the secular stagnation story. Chart 1 depicts the estimation of equilibrium real interest rates by Thomas Laubach, director of the Division of Monetary Affairs of Federal Reserve Board, and John C. Williams, President and Chief Executive Officer of the Federal Reserve Bank of San Francisco. This estimate is frequently referred to when discussing secular stagnation. Laubach and Williams (2003) assume a model comprised of a conventional backward-looking IS curve and Phillips curve (hereinafter referred to as “LW Model”) and use the Kalman filter to estimate the equilibrium real interest rates. According to the latest published data, equilibrium real interest rates were on a long-term declining trend and dropped sharply when the global financial crisis erupted in 2008.

But the estimation results based on the LW Model indicate that equilibrium real interest rates did not decline to the level required to strongly support the secular stagnation theory. Equilibrium real interest rates in the forth quarter of 2015 were negative 0.1%, and if the expected inflation rate is assumed to be 2%, equilibrium nominal interest rates can be calculated as nearly 2%. This means that the FRB could maintain accommodative conditions that were loose enough even after the interest rate hike in December last year, and this is not the kind of environment that invites secular stagnation. In addition, the estimation of equilibrium real interest rates accompanies a

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7 Update on March 3, 2016 based on Laubach and Williams (2003), hereinafter referred to “Laubach and Williams (2016)”.  

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large standard error that cannot be overlooked. According to our recalculation based on the program code and data provided by Director Laubach and President Williams, the standard error of equilibrium real interest rates is 2.0%. Hence, the current equilibrium real interest rates are in the range of ±2% and cannot be confirmed to be in negative territory.

![Chart 1: Equilibrium real interest rates by Laubach and Williams](image)

Note: Update on March 3, 2016 based on Laubach, Thomas and John C. Williams (2003). “Measuring the Natural Rate of Interest,” Review of Economics and Statistics, November. The dotted line represents the author’s recalculation. It does not exactly match the updated results by Laubach and Williams (2003) but discrepancy can be seen as negligible. Source: Made by MHRI based on data taken from the above paper.

3. Estimation results of the modified LW Model support the secular stagnation theory

The author recalculated after modifying the LW Model as follows. The results show that equilibrium real interest rates in the US are negative 2% and continue to be in the negative range even when taking the standard error into account.

What triggered the author to modify the LW Model was the GDP gap estimated and

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8 The original code functions in the software called GAUSS™. The author rewrote the code for another software called Eviews®, and updated the data (the provided data only covered the period up to April to June 2014) and conducted the recalculation. The dotted line in Chart 1 depicts the results of the author’s recalculation, and the results were confirmed to be almost the same as the published data.
published by Laubach and Williams(2016), along with equilibrium real interest rates. **Chart 2** shows the two types of the GDP gap published by Laubach and Williams (2016) and another GDP gap estimated by the Congressional Budget Office (CBO). In Chart 2, a thick line and a thin line show a filtered (one-sided) and a smoothed (two-sided) estimate respectively using the Kalman filter by Laubach and Williams (2016).

We can see throughout the 2000s that the GDP gap by the LW Model continued to be higher than the CBO-based GDP gap, and that a divergence existed between the two series of the LW Model.

According to the LW Model, the GDP gap in the US recorded a substantial plus before the outbreak of the global financial crisis. The drop at the time of the financial crisis was small compared with the estimate by the CBO, and the GDP gap has already returned to positive. In contrast, the CBO-based GDP gap was hovering around zero at best before the financial crisis and continues to remain in negative territory.

**Chart 2: Estimates of the GDP gap by LW model and CBO**

![Chart 2: Estimates of the GDP gap by LW model and CBO](image)

Note: A thick line and a thin line show a filtered (one-sided) and a smoothed (two-sided) estimate of the GDP gap respectively by Laubach and Williams (2016). The dotted line shows an estimate by CBO.

Source: Made by MHRI based on data taken from the above paper, the US Department of Commerce and CBO.

Also, observing the GDP gap based on the LW Model, we can see a large deviation emerge between the two series. The two series differ from the quantity of information applied in the estimation process. The smoothed estimate (thick line graph) is calculated from information taken from the entire sample period, and the filtered estimate is calculated only from the information beginning from the outset of the sample period up
until a specific period. A large deviation between two series implies that the GDP gap may be misread if the calculation overly depends on past information and that the model needs to incorporate some indicators related to future expectations.

It is a well-known problem as the “inflation puzzle” that the conventional Phillips curve employed by the LW Model cannot fully explain why the US economy did not slip into deflation despite the significantly negative GDP gap after the financial crisis. The LW Model seems to overcome this problem by, if not intentionally, estimating a higher GDP gap than that of the CBO.

Then, we focused on the Phillips curve in the LW Model and incorporated inflationary expectations of the future. To calculate the real federal funds rate, Laubach and Williams (2003, 2016) used ex-post inflation expectations estimated by the autoregressive model with order 3. So we incorporated this ex-post inflation expectations into the Phillips curve.

Chart 3 depicts the reestimated GDP gap. We can see that the deviation between a filtered and a smoothed estimate has been almost eliminated. Furthermore, while the reestimated figures also show a “better” gap than the CBO-based GDP gap, the GDP gap remains flat in negative territory for the recent years and moves closer to the CBO-based GDP gap than the original estimates by Laubach and Williams (2016).

According to our modified LW Model, equilibrium real interest rates in the US have reached negative 2%, demonstrating a wide difference from the estimate by the original LW Model (Chart 4). The standard error came in at 1.8%, an improvement from the conventional LW Model, and we can infer that equilibrium real interest rates in the US are within negative territory, even when taking into account the standard error.

The absolute value of log-likelihood is 298.6 for the conventional LW Model and 194.5 for our modified LW Model. Thus we can say that our modified LW Model with its smaller absolute value better explains the reality than the original LW Model (refer to the supplementary discussion for the parameter estimation results).

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9 Equilibrium real interest rates have two series, namely, the smoothed estimate and the filtered estimate, and this report refers to the filtered estimate.
**Chart 3: Reestimates of the GDP gap by the modified LW model**

Source: Made by MHRI based on the US Department of Commerce and CBO (the estimates are given by the author).

**Chart 4: Reestimates of equilibrium real interest rates and comparison with the LW model**

Note: The LW Model is based on published data, and the modified LW Model is based on the author's estimates.

Source: Made by MHRI.
4. Reestimated figures imply that the secular stagnation theory is valid

Our newly estimated equilibrium real interest rates suggest that the secular stagnation theory is valid as well as the need to create new demand.

According to our reestimated figures, equilibrium real interest rates show a substantial deterioration, falling to negative 2% with no signs of improvement. With the shock of the financial crisis, equilibrium real interest rates declined to nearly zero percent and declined even further around 2012. In the original and modified LW Models, equilibrium real interest rates are defined as a sum of the potential GDP growth rate and demand shock. The recent change in equilibrium real interest rates was brought about by a significant deterioration of demand shock, and the contribution of the potential GDP growth rate proved to be limited (Chart 5).

![Chart 5: Factors behind the fluctuation of equilibrium real interest rates](image)

Note: All of the graphs above are based on the author's estimates. Potential GDP growth rates are on a contribution basis. Source: Made by MHRI.

If we look back at the past movement of equilibrium real interest rates, we can see there was another period when interest rates worsened to nearly zero percent during the mid-1990s. The factor behind the deterioration at that time was also the worsening of demand shock. But equilibrium real interest rates recovered to the range of 2% to 4% by

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the year 2000 as demand shock improved. At that time, the US federal budget were going into the black, so it seems that fiscal expansion, which is one of the important prescription to secular stagnation by Mr. Summers, was not the reason behind the improving demand shock. Monetary policy under Mr. Greenspan was also tight, which is clear from Chart 4. Applying the process of elimination, we may infer that the improvement of demand shock was by the IT boom at that time, not by policy responses for creating an aggregate demand.

The current equilibrium real interest rates are substantially negative. We hold that the FOMC should stop raising the policy interest rate and wait for equilibrium real interest rates to recover. Moreover, caution is required on the decline of inflationary expectations nowadays. The inflation expectations employed in the LW Model remained low in the 1.5% range, and the FRB chair Yellen herself expressed strong concern over the decline of inflation expectations in the survey research at a recent speech (March 29).

The recovery of equilibrium real interest rates is left up to the recovery of demand shock. The mid-1990s’ experience suggested that an aggregate demand policy is not always needed. But with several years of stagnation already, if not secular, we can not afford to wait for another innovation boom. To realize this recovery, we believe the US government should create aggregate demand through fiscal policies, strengthen the income redistribution to economic units with higher expenditure propensities by tax policy, and encourage market competition even further.
Supplementary discussion: Outline and estimate results of the original and our modified LW Models

(1) LW Model

**IS curve**
\[ \bar{y}_t = A_y(L)\bar{y}_{t-1} + A_r(L)(r_{t-1} - r^*_t) + \epsilon_{1t} \]

**Phillips curve**
\[ \pi_t = B_{\pi}(L)\pi_{t-1} + B_y(L)\bar{y}_{t-1} + B_x(L)x_t + \epsilon_{2t} \]

**GDP gap**
\[ \bar{y}_t = y_t - y^*_t \]

**Potential GDP**
\[ y^*_t = y_{t-1}^* + g_t + \epsilon_{3t} \]

**Potential GDP growth rate**
\[ g_t = g_{t-1} + \epsilon_{4t} \]

**Equilibrium real interest rate**
\[ r^*_t = c_{gt} + z_t \]

**Demand shock**
\[ z_t = z_{t-1} + \epsilon_{5t} \]

**Real interest rates**
\[ r_t = i_t - \pi_{t|AR(3)} \]

(2) Modified LW Model (only the modified item)

**Phillips curve**
\[ \pi_t = B_{\pi}(L)\pi_{t-1} + B_y(L)\bar{y}_{t-1} + B_x(L)x_t + \epsilon_{2t} \]

\[ A_y(L), A_r(L), B_{\pi}(L), B_y(L), B_x(L) \] are lag operators related to the variables of subscript letters. \( x_t \) is an explanatory variable of the Phillips curve other than GDP gap. \( i_t \) represents policy interest rate, and \( \pi_{t|AR(3)} \) is ex-post inflation expectations based on the autoregressive model with order 3.

Both models carry the restriction that the sum of the parameters for Phillips curve’s inflation expectations and the past inflation must equal one. Also, variance of the respective shocks (\( \epsilon_{3t}, \epsilon_{4t}, \epsilon_{5t} \)) concerning potential GDP, potential GDP growth rate, and demand shock are calculated using the median-unbiased estimator. The estimation period of the Kalman filter is 1961 to 2015 (quarterly data).
<table>
<thead>
<tr>
<th>Parameters</th>
<th>LW Model</th>
<th>Estimated value</th>
<th>Standard error</th>
<th>Modified LW Model</th>
<th>Estimated value</th>
<th>Standard error</th>
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**Standard error of potential variables (average)**

- $y_t$  
  - $\gamma$: 2.06864  
  - $\gamma$: 0.54587
- $g_t$:  
  - $\gamma$: 0.29793  
  - $\gamma$: 0.35381
- $r_t^*$:  
  - $\gamma$: 2.01685  
  - $\gamma$: 1.76635

Note: Both LW Model and modified LW Model are calculated by the author.

Oil and imp represent the increase rate of crude oil price and import goods price (excluding crude oil) relative to core inflation.

$S$ represents the standard error concerning the shocks of subscript letters.

Source: MHRI